IMPACT OF A LIQUID ON THE INCLINED WALL OF AN INFINITE, PARTLY CLOSED CONTAINER

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Westergardt [1] had solved a problem of the impact of an incompressible liquid on a vertical weir by expansion into series, in 1931.

In 1956, Napetvaridze [1] tried to extend the Westergardt method to the inclined weir, but made an error in his solution. The problem of the impact of incompressible liquid on an inclined wall in the presence of an adjacent cover is solved in this paper, applying the theory of functions of a complex variable. The obtained solution can be applied to the study of the impact of liquid on an inclined dam or the impact of liquid cargo on a bulkhead of a ship.

1. Statement of the problem. The following plane problem is discussed here. An infinite container of finite depth h is filled with an incompressible non-viscous liquid. The side wall AB is inclined to the horizontal plane bottom AC at an angle $\pi\alpha$. The top is covered by a plate BD, adjacent to the side wall, of length λh, (where $0 < \lambda < \infty$). The free surface is DC_1 in the vertical projection (Fig.1).

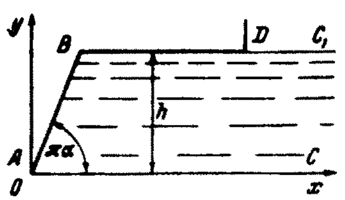

Fig. 1.

Assume, that after being at rest, the container suddenly obtains a velocity v_0 in a horizontal direction. The force exerted by the liquid on the inclined wall AB is to be determined. The plane of flow is considered as a plane of the complex variable $z = x + yi$; the axis x coincides with the bottom of container, the coordinate center is at the intersection of the wall and bottom (Fig.1). The impulsively started flow of liquid has a velocity potential ϕ, which is related to the impulsive

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pressure p and the density of liquid ρ by the expression (2):

$$p = -\rho\varphi \tag{1.1}$$

Thus, the problem is reduced to the determination of the velocity potential ϕ of the real part of the complex potential of flow $w = \phi + \psi i$ with the following boundary conditions: the pressures at the free surface equal zero, and the normal velocities of the liquid at the walls $\partial\phi/\partial n$ coincide with the normal velocities of the walls v_n :

$$\varphi = \operatorname{Re} w = 0 \quad \text{on } DC_1, \qquad \frac{\partial\varphi}{\partial n} = v_n \quad \text{on } AB,\ AC,\ BD \tag{1.2}$$

2. Solution of the problem. The flow field z, represented by the open triangle with angles $A = \pi\alpha$, $B = \pi(1 - \alpha)$ and $C = 0$, is mapped on the

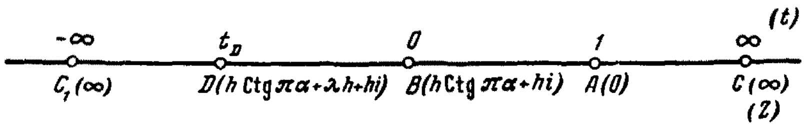


Fig. 2.

upper semi-plane of the parametric variable t. Fig. 2 shows the relationship between points in the physical and the transformed planes. Thus (3):

$$z = A\int_1^t \frac{dt}{(t-1)^{1-\sigma}t^{\alpha}}, \qquad \frac{dz}{dt} = \frac{A}{(t-1)^{1-\alpha}t^{\alpha}} \tag{2.1}$$

In order to find $\phi = \operatorname{Re} w$, the value of $dw/dt = (dw/dz)(dz/dt)$ is to be determined by the boundary conditions. On the part of the real t axis which corresponds to the inclined wall AB, the cover BD and the bottom AC, one has the relation (4):

$$\operatorname{Im}\frac{dw}{dt} = -\frac{\partial\varphi}{\partial n}\left|\frac{dz}{dt}\right| \tag{2.2}$$

from which follows:

(1) For the part of the t axis which corresponds to the bottom AC and the cover BD, $v_n = 0$; therefore, conforming to (1.2) and (2.2):

$$\operatorname{Im}\frac{dw}{dt} = 0 \qquad \text{for } 1 < t < \infty,\ t_D < t < 0 \tag{2.3}$$

(2) For the part of the t axis which corresponds to the side wall AB (Fig.1), if follows from conditions (1.2), (2.1) and (2.2) that

$$\operatorname{Im}\frac{dw}{dt} = -v_0\sin\pi\alpha\left|\frac{dz}{dt}\right| = \frac{-Av_0\sin\pi\alpha}{(1-t)^{1-\alpha}t^{\alpha}} \qquad \text{for } 0 < t < 1 \tag{2.4}$$

(3) For the part of the t axis which corresponds to the free surface DC_1 , according to (1.2):

$$\operatorname{Re}\frac{dw}{dt} = 0 \qquad \text{for } t < t_D \tag{2.5}$$

A function $f(t)$ is introduced for the determination of dw/dt:

$$f(t) = \frac{dw}{dt}\sqrt{t - t_D} \qquad (t_D < 0) \tag{2.6}$$

The boundary conditions for $f(t)$ are obtained by the transformation of the boundary conditions for dw/dt:

$$\operatorname{Im} f(t) = \operatorname{Im} \frac{dw}{dt}\sqrt{t-t_D} = 0 \qquad \text{for } 1<t<\infty$$

$$\operatorname{Im} f(t) = \operatorname{Im} \frac{dw}{dt}\sqrt{t-t_D} = -\frac{v_0 A \sin\pi\alpha}{(1-t)^{1-\alpha}t^{\alpha}}\sqrt{t-t_D} \qquad \text{for } 0<t<1$$

$$\operatorname{Im} f(t) = \operatorname{Im} \frac{dw}{dt}\sqrt{t-t_D} = 0 \qquad \text{for } t_D<t<0$$

$$\operatorname{Im} f(t) = \operatorname{Re} \frac{dw}{dt}\sqrt{|t|-|t_D|} = 0 \qquad \text{for } t<t_D \tag{2.7}$$

The function $f(t)$ is determined from the values of its imaginary part given on the entire real axis.

$$f(t) \doteq \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im} f(\xi)\,d\xi}{(\xi-t)} \tag{2.8}$$

Hence, taking in account (2.7):

$$f(t) = -\frac{Av_0 \sin\pi\alpha}{\pi}\int_0^1 \frac{\sqrt{\xi-t_D}\,d\xi}{(1-\xi)^{1-\alpha}\xi^{\alpha}(\xi-t)} \tag{2.9}$$

It follows from (2.9) and (2.6):

$$\frac{dw}{dt} = \frac{f(t)}{\sqrt{t-t_D}} = \frac{-Av_0\sin\pi\alpha}{\pi\sqrt{t-t_D}}\int_0^1 \frac{\sqrt{\xi-t_D}\,d\xi}{(1-\xi)^{1-\alpha}\xi^{\alpha}(\xi-t)} \tag{2.10}$$

Integration on t along any contour gives, by-passing the singular point and determining the arbitrary constant from the condition $\phi(t) = \operatorname{Re} w(t) = 0$ when $t = t_D$:

$$\varphi(t) = \frac{-2Av_0\sin\pi\alpha}{\pi}\int_0^{\pi/2} (\operatorname{tg}\theta)^{1-2\alpha} \ln\left|\frac{\sqrt{t-t_D}+\sqrt{\sin^2\theta-t_D}}{\sqrt{t-t_D}-\sqrt{\sin^2\theta-t_D}}\right| d\theta \tag{2.11}$$

where $\xi = \sin^2\theta$.

The pressure at a point of the side wall is obtained after substitution of $\phi(t)$, taken from (2.11), into (1.1):

$$p(t) = \frac{2A\rho v_0\sin\pi\alpha}{\pi}\int_0^{\pi/2} (\operatorname{tg}\theta)^{1-2\alpha} \ln\left|\frac{\sqrt{t-t_D}+\sqrt{\sin^2\theta-t_D}}{\sqrt{t-t_D}-\sqrt{\sin^2\theta-t_D}}\right| d\theta \tag{2.12}$$

The constant A is determined from (2.1), because

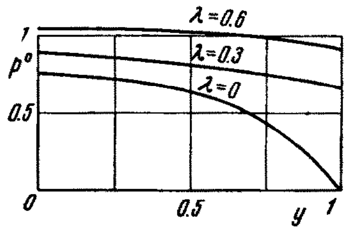
$$y = \text{IM } z = h \quad \text{when} \quad t = 0$$

The relationship of t_D with λ, where λh is the length of the cover (Fig.1), can be determined from (2.1).

Parametric equations of the curve of pressure variation along the side walls can be easily determined from (2.12) and (2.1). Approximate computations were performed for the following particular cases, using

α	1/2 (Fig. 3)			1/4 (Fig. 4)			3/4 (Fig. 5)		
t_D	0	−1/2	−1	0	−1/3	−1	0	−1/3	−1
$\lambda \approx$	0	0.3	0.6	0	0.2	0.3	0	0.9	1.2
$p_{max}/\rho v_0 h$	0.74		1.05	1.19		1.42			1.25
$p_{min}/\rho v_0 h$	0		0.94	0		0.66			0.72

The graphical representation of p as a function of y, based on these approximate computations, is shown on Figs. 3, 4 and 5, taking $p^0 = p/\rho v_0$



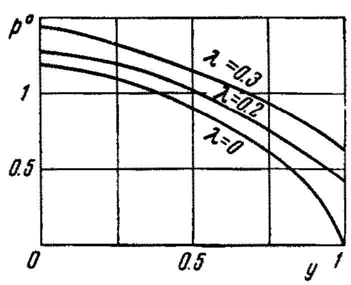


Fig. 4.

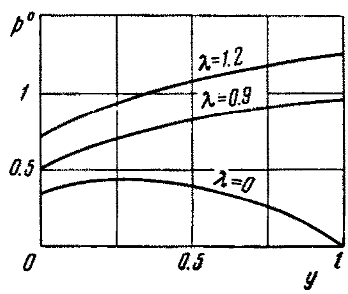


Fig. 5.

Thus, the cover substantially increases the impulsive forces acting on the side wall. The influence of the angle of inclination of the side wall is clear from the comparison of Figs. 3, 4 and 5.

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